Gripping a Kitchen Knife from the Cutting Board

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Abstract—Despite more than three decades of grasping research, many tools in our everyday life still pose a serious challenge for a robotic hand to grip. The level of dexterity for such a maneuver is surprisingly “high” that its execution may require a combination of closed loop controls and finger gaits. This paper studies the task of an anthropomorphic hand driven by a robotic arm to pick up and firmly hold a kitchen knife lying on the cutting board. In the first phase, the hand grasps the knife’s handle at two antipodal points and then raises it using the knife’s point on the board as a pivot to leverage the latter’s support. Desired contact forces exerted by the two holding soft fingers are calculated and used for dynamic control of both the hand and the arm. In the second phase, a sequence of gaits for all the fingers are performed quasi-statically to reach a power grasp on the knife’s handle, which remains still during the period. Simulation has been performed using models of the Shadow Hand and UR10 Arm.

I. INTRODUCTION

Human-level dexterity has long been a grand challenge and a lofty objective to drive robotics research. A convincing evidence would be robots having the ability to manipulate tools that are used in everyday life. Such robots will be able to assist us effectively, from housework to health care to medicine to manufacturing, and even to scientific exploration. The ability of using tools is a big step for a robot to become “general purpose”.

A robot today does not know how to handle a hand tool properly, whether to mount a wrench on a nut, to move the tip of a screwdriver into a groove on a screw, or to grip a kitchen knife and cut a food item into pieces. Each mentioned skill involves delicate movements of which automation must rely on force/position feedback and finger gaits. We should exploit the underlying mechanics and use proper control policies for coordinating the robotic arm, hand, and fingers. Comprising a skillful maneuver are usually multiple elementary actions, some of which can be performed quasi-statically for their low speeds while others are better controlled dynamically in a feedback driven way for error correction and stability.

In this paper, we study how a pair of robotic arm and hand picks up a kitchen knife lying on the cutting board and hold its handle tightly to be ready for food cutting. The arm is assumed to have at least six degrees of freedom (DOFs), and the hand is assumed to be anthropomorphic. However, the gripping strategy to be introduced can be carried out with any hand-arm pair that have considerably fewer number of DOFs. Every finger of the hand is modeled as a soft finger, which can exert a moment about the contact.

Fig. 1(a) illustrates the four steps of the action. The hand first places two fingers at antipodal positions on the knife’s handle. Next, leveraging the board’s support with the knife’s point as a pivot, the hand raises the handle to let the knife rotate under gravity about the two finger contacts (Fig. 1(b)). The amount of squeeze during this lift needs to be kept at a proper level to allow the knife’s rotation about the pivot and yet prevent a loss of either finger contact on the handle. The arm’s
movement, in the meantime, is controlled to coordinate with the knife’s rotation about the pivot. The dynamics of the knife, hand, and arm are combined, while control of the arm is decoupled from that of the hand via the use of a force sensor connecting them. Keeping the knife in balance with the palm involved, finger gait and palm movements are carried out quasi-statically on the handle to form a power grasp. This is shown in Fig. 1(c) and (d).

II. RELATED WORK

Gripping of a kitchen knife is a dexterous maneuver with that contains versatile elements of human dexterity long sought for robots. Swift moves used by the human hand to accomplish this task challenges our understanding about dexterous manipulation. This dynamic manipulation task [1] can be decomposed into a sequence of primitive actions, which are non-prehensile and executed either dynamically or quasi-statically.

Recent works [2], [3] analyzed and categorized human grasping behaviors. Meanwhile, human demonstration has remained an important source for robots to acquire skills in assembly [4], [5], dynamic manipulation [6], and tool manipulation [7].

In more than three decades, research related to robotic hands has been conducted on rigid body grasping [8], [9], in-hand manipulation [10], [11] with contacts of rolling [12], [13] and sliding [14], [15], [16], and manipulation using dynamic [17], [18] and impulsive [19], [20], [21] forces. Dexterous manipulation has been investigated extensively [22], [23], [24] but mostly applied to real and synthetic objects with simple geometry.

Preliminary progress has been made on robotic manipulation of hand tools to accomplish tasks such as drilling and pencil drawing [25], sausage pickup and bottle opening[26], bolt unscrewing with a wrench [27]. These works, carried out with visual guidance, nevertheless, did not leverage task mechanics with no consideration for factors such as compliance, friction, contact modes.

Though proposed almost three decades ago [28], finger gaiting has remained more a subject for theoretical inquiry with simulation using unrealistically simplified hand models [29], [30].

Kitchen knife gripping is a highly contact-based tasks in which force feedback and force control can be important for achieving a robust performance [31]. To deal with contact constraints, controls of force and position are more effectively carried out in the workspace [32]. Hybrid force/position control [33] is effective in the first phase of lifting up the knife’s handle while regulating the cutting board’s supporting force on the knife’s tip.

III. PIVOTING OF THE KNIFE

A kitchen knife with known geometry and mass property lies on the cutting board. The knife has a plane II of symmetry which cuts it into two mirrored halves, intersecting it at a planar region bounded by a contour $\gamma$ and containing the knife’s point $p$ and center $c$ of mass.

The robotic hand will start with placing its thumb and index finger at a pair of antipodal points $p_1$ and $p_2$ (see Fig. 2) on the handle portion of the contour $\gamma$. More specifically, the inward normals $\hat{n}_1$ and $\hat{n}_2$ of $\gamma$ at $p_1$ and $p_2$ are opposite to each other, and collinear with the line $\ell$ through these two points, referred to as the antipodal line. Since $\gamma$ is in the plane of symmetry, the two points must also be antipodal on the knife’s handle.

A body frame $\{B\}$ of the knife is located at $c$ with its $x$-axis, referred to as the $x_B$-axis, in the direction from $p_2$ to $p_1$, and its $x_By_B$-plane coinciding with II.

A. Path of the Knife Pivoting

A world frame $\{W\}$ is located on the cutting board at the point $p_0$ in contact with the knife’s point, as shown in Fig. 3(a). Its $xy$-plane is aligned with the board surface. Denote by $K_0$ the knife’s initial pose. Fig. 3(a) also shows an intermediate pose $K_1$ in which the knife’s point stays at $p_0$ but its handle is raised such that

1) its center of mass $c$ stays in the same vertical plane

as that in the pose $K_0$, and

2) the antipodal line $\ell$ is parallel to the board surface.

The angle $\phi$ of rotation by the vector $c - p_0$ from its initial position is set to a small value. The $x_W$-axis is in the direction from $p_1$ to $p_2$, i.e., that of the $x_B$-axis in the pose $K_1$. Denote by $\hat{x}$, $\hat{y}$, and $\hat{z}$ the unit vectors on the $x_W$-, $y_W$-, and $z_W$-axes.

Fig. 3(b) displays the final pose $K_2$ of pivoting, which results from rotating the knife in the pose $K_1$ about the $x_W$-axis until the handle’s end is contact with the palm. The angle $\psi$ of this rotation can be calculated in advance. Linear interpolations are used for constructing

1We refer to [34] for computation of all antipodal points on a closed plane curve.
a path of the knife from $K_0$ to $K_1$ as a screw motion and another path from $K_1$ to $K_2$ as a rotation in terms of $\phi$. The two paths are then parameterized with time to become a single motion trajectory. The knife’s position and orientation at time $t$ are $c(t)$ and $R(t)$, respectively. They will be used as the desired position and orientation for control purpose later on, and from now on denoted as $c_d(t)$ and $R_d(t)$.

B. Initial Finger Placement

For convenience, we also refer to the thumb, and the index, middle, ring, and little fingers as finger 1, 2, 3, 4, 5, respectively. Finger $i$ has joint angle vector $\theta_i$. Its tip is modeled by a smooth curved (e.g., ellipsoidal) surface $\sigma_i(u_i, v_i)$ in its local frame. In the world frame $\{W\}$, the surface of finger $i$’s tip in its current pose results from a rigid transformation of $\sigma_i(u_i, v_i)$ determined by $\theta_i$ and $\theta_a$, the joint angle vector of the arm. Denote the surface by $\sigma_i[\theta_i, \theta_a](u_i, v_i)$.

The thumb and index finger are to be placed at $p_1$ and $p_2$ where their outward surface normals must coincide with the normals $n_1$ and $n_2$ on the handle, respectively. This reduces to finding the values of $\theta_a, \theta_1, u_i, v_i$, $i = 1, 2$ to satisfy the following ten equations:

$$\sigma_i[\theta_i, \theta_a](u_i, v_i) = p_i,$$

$$\hat{t}_{ij} \cdot \left( \frac{\partial \sigma_i}{\partial u_i} \times \frac{\partial \sigma_j}{\partial v_i} \right) = 0, i, j = 1, 2,$$

where $\hat{t}_{ij}$, $j = 1, 2$, are two orthogonal tangent vectors to $\sigma_i$ at $p_i$.

A subspace of solutions exists given that the total number of degrees of freedom of the arm and two fingers far exceeds ten. To visualize one process finding a solution, we can keep the palm stationary while letting the two fingers close in until their distance reaches $\|p_1 - p_2\|$. The distance is achieved at the closest pair of points on different fingertips. The outward normals at these two points are collinear, which directly follows from vanishing of the partial derivatives of $\|p_1 - p_2\|^2$ with respect to $u_1, v_1, u_2$, and $v_2$. The arm then moves the hand until these two points and their outward normals coincide with $p_1$ and $p_2$ and their inward normals, respectively.

C. Knife Dynamics

As shown in Fig. 1(a), the hand, with its thumb and index finger placed on the handle, rotates the knife using its tip located at $p_1$ as a pivot. The tip receives a supporting force $f_0$ from the cutting board. Negligible frictional force on the tip implies $f_0 = f_0 \hat{z}$, for some $f_0 \geq 0$.

Let $m$ be knife’s mass and $Q$ be its inertia tensor expressed in $\{W\}$:

$$\sum_{i=0}^{2} f_i + mg = m\ddot{c},$$

where $g$ is the gravitational acceleration. Each finger is a soft finger which exerts a moment about the contact normal with a magnitude upper-bounded by a ratio $\eta$ times the normal contact force [35, p. 219]. This ratio is referred to as the torsional torque coefficient. Both fingers exert moments that achieve this ratio when the knife’s handle is rotating about the antipodal line $\ell$. In this situation, Euler’s equation is given as

$$\sum_{i=0}^{2} r_i \times f_i - \eta \sum_{j=1}^{2} (f_j \hat{n}_j) \hat{n}_j = Q\dot{\omega} + \omega \times Q\omega,$$

where $\omega$ is the knife’s angular velocity in the world frame $\{W\}$.

D. Desired Finger Forces for Pivoting

Let us now calculate the desired contact forces $f_i$, $i = 0, 1, 2$, provided by the board and two fingers to carry out the pivoting about $p_0$ as described in section III-A. They will be used as desired values for control shortly. Let $r_0$ and $B r_0$ be the vector from the knife’s center $c$ of mass

$$Q = R_k Q_k R_k^T,$$

where $Q_k$ is the diagonalized inertia tensor in the frame defined by the knife’s principal axes, and $R_k$ is the matrix describing the rotation of this frame from $\{W\}$. 

Fig. 3: Knife’s initial, intermediate, and final poses $K_0, K_1, K_2$ during pivoting. (a) The $x_W$-axis, which is parallel to the antipodal line $\ell$ in $K_1$, and (b) rotation along this axis.
force. Otherwise, we reparameterize the knife’s pose to half of the magnitude of the knife’s gravitational force, and choose one from (4) with \( f \in f \). Under this condition, the contact force generates a moment \( m \) due to the gravitational \( \omega \times r_0 \). Thus, the knife’s handle must be oriented to support the torque due to the gravitational \( \omega \times r_0 \). The solution (4) subject to three types of constraints: First, the board contact force must be positive \( f > 0 \) and \( f > 0 \) for the knife’s tip to become a pivot:

\[ f_0 > 0. \]

The linear system (3) has a one-dimensional subspace of solutions:

\[ [f_0 \ f_1^\top \ f_2^\top]^\top = f^* + \lambda \tilde{f}, \]

where \( f^* \) is the least-square solution and \( \tilde{f} \) is a vector in the null space of the \( 6 \times 7 \) coefficient matrix.

E. Finger and Arm Dynamics

For knife pivoting, the middle, ring, and little fingers are not used and hence not considered. Denote by \( \theta_h = (\theta_1^h, \theta_2^h)^\top \) the hand configuration. The dynamics of the thumb, index finger, and arm are combined:

\[ \begin{bmatrix} \tau_a \\ \tau_h \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \begin{bmatrix} \tilde{\theta}_a \\ \tilde{\theta}_h \end{bmatrix} + \begin{bmatrix} N_1 + J_{1a}^h f \\ N_2 + J_{2a}^h w \end{bmatrix} \]

where \( \tau_a \text{ and } \tau_h \) are the joint torques by the arm and hand, \( N_1 \text{ and } N_2 \) includes the gravitational, Centrifugal and Coriolis terms, \( w = (w_1^T, w_2^T)^\top \) stacks the external wrenches applied to the two fingers, and \( (J_1, J_2) \in \mathbb{R}^{12 \times (k_a + k_b)} \), with \( k_a \text{ and } k_b \) being the DOFs of the arm and the two fingers, together represents the robot’s Jacobian evaluated at the two contact points. We rewrite (8) in the following form:

\[ \begin{bmatrix} \tau_a \\ \tau_h \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{bmatrix} \begin{bmatrix} \tilde{\theta}_a \\ \tilde{\theta}_h \end{bmatrix} + \begin{bmatrix} N_1 + J_{1a}^h f \\ \tilde{N}_h + J_{2a}^h f \end{bmatrix}, \]

where

\[ \tilde{N}_h = N_2 - M_{21} J_{1a}^{-1} N_1 + M_{21} M_{11}^{-1} \tau_a, \]

\[ \tilde{J}_h = J_2 - J_1 M_{11}^{-1} M_{21}. \]

The hand dynamics, as the second row of equations in (9), contain the arm’s state (joint angles \( \theta_a \) and velocities \( \dot{\theta}_a \)) as well as its torques \( \tau_a \) due to coupling between the arm and hand. The arm’s joint accelerations \( \ddot{\theta}_a \) have been eliminated, which means that we can have the full hand dynamics once knowing the arm and hand’s states and the former’s joint torques \( \tau_a \).

Some anthropomorphic hands are tendon driven and the joints are coupled under linear holonomic constraints \( h(\theta_h) = 0 \). In this situation, the hand dynamics need to be rewritten in terms of independent generalized coordinates \( q_h \), based on the relationship \( \dot{\theta}_h = V \dot{q}_h \), for some matrix \( V \). This will transform the hand dynamics in (9) to

\[ \tau_h = M_h \dot{q}_h + N_h + J_h^T \dot{w}, \]

where \( M_h = V^T \tilde{M}_h V, \ N_h = V^T \tilde{N}_h, \text{ and } J_h = V^T \tilde{J}_h. \)

To the arm, the hand can be treated as an external workload measurable by a force/torque sensor mounted between the arm’s end-effector and the hand. With the sensor reading \( \dot{w}_s \in \mathbb{R}^6 \), the arm’s dynamics become

\[ \tau_a = M_a \dot{q}_a + N_a(q_s, \dot{q}_a) + J_s^T \dot{f}_s, \]

where \( \tau_a \text{ and } \tau_h \) are the joint torques by the arm and hand, \( N_1 \text{ and } N_2 \) includes the gravitational, Centrifugal and Coriolis terms, \( w = (w_1^T, w_2^T)^\top \) stacks the external wrenches applied to the two fingers, and \( (J_1, J_2) \in \mathbb{R}^{12 \times (k_a + k_b)} \), with \( k_a \text{ and } k_b \) being the DOFs of the arm and the two fingers, together represents the robot’s Jacobian evaluated at the two contact points. We rewrite (8) in the following form:

\[ \begin{bmatrix} \tau_a \\ \tau_h \end{bmatrix} = \begin{bmatrix} M_{11} & M_{12} \\ 0 & M_{22} \end{bmatrix} \begin{bmatrix} \tilde{\theta}_a \\ \tilde{\theta}_h \end{bmatrix} + \begin{bmatrix} N_1 + J_{1a}^h f \\ \tilde{N}_h + J_{2a}^h f \end{bmatrix}, \]

where

\[ \tilde{N}_h = N_2 - M_{21} J_{1a}^{-1} N_1 + M_{21} M_{11}^{-1} \tau_a, \]

\[ \tilde{J}_h = J_2 - J_1 M_{11}^{-1} M_{21}. \]

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where \( M_h = V^T \tilde{M}_h V, \ N_h = V^T \tilde{N}_h, \text{ and } J_h = V^T \tilde{J}_h. \)

In the rest of the paper, we will use \( q_a = \theta_a \) and \( q_h \) to denote the robot configuration.

F. Arm Trajectory and Control

To the arm, the hand can be treated as an external workload measurable by a force/torque sensor mounted between the arm’s end-effector and the hand. With the sensor reading \( \dot{w}_s \in \mathbb{R}^6 \), the arm’s dynamics become

\[ \tau_a = M_a \dot{q}_a + N_a(q_s, \dot{q}_a) + J_s^T \dot{f}_s, \]
where $M_a$ is the arm’s mass matrix, $N_a$ includes the nonlinear terms, and $J_a$ is the arm’s Jacobian matrix at the reference point of the sensor.

The arm is responsible for moving the palm along its desired trajectory to realize pivoting performed by the thumb and index finger. Denote by $x_a \in \mathbb{R}^6$ the pose of the arm’s end-effector as determined by its location $p_a$ and orientation matrix $R_a$,3 where

$$P_a(t) = \begin{cases} R(t)R(0)^\top p_a(0), & t \in [0, t_1], \\ p_a(t_1) + p_i(t) - p_i(t_1), & t \in (t_1, t_2], \\ \end{cases}$$

$$R_a(t) = \begin{cases} R(t)R(0)^\top R_a(0), & t \in [0, t_1], \\ R_a(t_1), & t \in (t_1, t_2]. \\ \end{cases}$$

The desired trajectory $x_{a,d}$ of the arm can be constructed eventually from the desired trajectory of the knife.

Let $J_a$ be the Jacobian matrix of the arm for $x_a$. We have $\dot{x}_a = J_a \dot{q}_a$ and $\ddot{x}_a = J_a \ddot{q}_a + J_a \dot{q}_a$. Assuming that the arm is not at a singular configuration, we obtain $\ddot{q}_a = J_a^\dagger (\ddot{x}_a - J_a \dot{q}_a)$, where $J_a^\dagger$ is the Penrose-Moore pseudoinverse of $J_a$. To control the arm, we make use of the error $x_{a,c} = x_{a,d} - x_a$. From the arm dynamics (11), a task space position controller is then employed as follows:

$$\tau_{a,ctrl} = M_a J_a^\dagger \left(\alpha - \dot{J}_a \ddot{q}_a\right) + N_a + J_a^\top f_a, \quad (12)$$

where

$$\alpha = \ddot{x}_{a,d} + K_{a,d} \ddot{x}_{a,c} + K_{a,p} x_{a,c} + K_{a,i} \int x_{a,c} \, dt,$$

and $K_{a,d}, K_{a,p}, K_{a,i}$ are the damping, proportional and integral gains, respectively.

**G. Finger Control**

The knife’s desired trajectory $(c_i(t), R_d(t))$ is determined in section III-A. For $i = 1, 2$, at time $t$ along the trajectory there is a desired contact frame $\{F_{id}\}$ on the tip of finger $i$ that is located at $p_i$ as determined from $(c_i(t), R_d(t))$ with its $z$-axis aligned with the contact normal $\hat{n}_i$ and $y$-axis parallel to $R_d(t)^\top \hat{z}$.

During the pivoting action, the actual trajectory of $p_i$ is

$$p_i(t) = R(t)(B r_i - B r_0), \quad t \in [0, t_2], \quad i = 1, 2,$$

where $B r_i$ is the vector from $c$ to $p_i$ in the knife’s body frame. The actual contact frame on the fingertip is $\{F_i\}$, which coincides with the (desired) frame $\{F_{id}\}$ at $t = 0$ (Fig. 4).

In order to let the knife track the desired pivoting trajectory, the forces exerted by the fingers should be exactly the same as obtained from section III-D. Instead of controlling all the contact forces directly, we let the index finger follow its desired trajectory under position control, and the thumb apply the desired normal contact force while following the trajectory in other dimensions. When all the contacts with the knife are maintained with no slip, it can be verified from the dynamics that its translational and rotational accelerations will stay the same as their desired values. In a sense, the normal contact force by the thumb determines the other two contact forces by the index finger and the board.

At time $t$ along the real knife trajectory, we let $x_i(t) \in \text{SE}(3)$ represent the transformation from the desired contact frame $\{F_{id}\}$ to the actual contact frame $\{F_i\}$. Since finger $i$ has $k_i < 6$ degrees of freedom, its configuration can be uniquely determined by a subset $s_i = (s_{iv}, s_{if})$ of coordinates in $x_i(t)$, where position and force control are applied on the coordinate subsets $s_{iv}$ and $s_{if}$, respectively. We introduce a $k_i \times 6$ selection matrix $S = (S_{iv}^\top, S_{if}^\top)^\top$ such that $S_{iv} = S_{iv} x_i$ and $s_{if} = S_{if} x_i$.

The index finger is subject to position control only, thus, $s_{2v} = s_{2v}$. The thumb has the $z$-component of $x_1(t)$ selected for force control while the other $k_1 - 1$ for position control. Note that the directions for its motion control and force control are orthogonal to each other, which leads to $(0, 0, 1, 0, 0, 0) S_{1v}^\top = 0$. By setting large enough gains for the index finger, we may assume that this finger and the knife’s handle together constitute a hard environment for the thumb to be in contact with. Consequently, the position along the $z$-direction of the task frame $\{P_s\}$ is fixed and $s_{if} = 0$ always holds. In fact, $s$ is now completely determined by $s_x$

$$s_1 = S_1 S_1^\top s_{1v}. \quad (13)$$

Let $q_i$ be the independent generalized coordinates of the $i$-th finger. For $i = 1, 2$, $s_i = H_i q_i$, where $H_i = S_i T_i J_{c,i}$, where $T_i$ transforms linear and angular
velocities from the world frame \( \{W\} \) to the change rate of \( \dot{x}_i \) in the task frame \( \{P_i\} \), and \( J_{c,i} \) is the finger Jacobian evaluated at the contact point \( p_i \). The \( k_i \times k_i \) matrix \( \hat{H}_i \) is invertible if the finger joints are not in a singular configuration. Differentiation of \( \dot{s}_i = H_i \ddot{q}_i \) yields, for \( i = 1, 2, \)

\[
\dot{s}_i = \dot{H}_i \ddot{q}_i + H_i \dddot{q}_i,
\]

from which we obtain

\[
\dddot{q}_i = \hat{H}_i^{-1} (\dot{s}_i - \dot{H}_i \ddot{q}_i).
\]

Plugging the last equation and (13) into the finger dynamics (10), we have the task space hand dynamics

\[
\tau_h = M_h \left[ H_1^{-1} \left( S_1 \dot{s}_1 + \dot{H}_1 \ddot{q}_1 \right) \right] + N_h + J_h^T \tau \tag{14}
\]

The hand controller is described as

\[
\tau_{h,ctrl} = M_h \left[ H_1^{-1} \left( S_1 \dot{s}_1 + \dot{H}_1 \ddot{q}_1 \right) \right] + N_h + \left[ J_1^T R_1 \dot{\beta}_1 \right] + J_h^T \tau \tag{15}
\]

where, denoting by \( s_{iv,d} \) the desired value of \( s_{iv} \) and \( s_{iv,e} = s_{iv,d} - s_{iv} \),

\[
\begin{align*}
\alpha_i &= \ddot{s}_{iv,d} + K_{f,d} \dot{s}_{iv,e} + K_{f,p} s_{iv,e}, \\
\beta_i &= w^b_{e} + K_{f,i} \int w^b_{e} dt,
\end{align*}
\]

\( w \) is sensed contact wrench, \( w^b_{e} \) is the force error along the selected directions in the contact frame \( \{F_{1d}\} \), \( N_h \) is the nonlinear term calculated based on the arm controller output \( \tau_{a,ctrl} \).

IV. POWER GRASP VIA FINGER GAITS

In this section, we introduce a manipulation strategy that follows pivoting to achieve a power grasp, which allows it to resist external disturbance in any directions. The knife can be repositioned and reoriented by the arm and hand as desired.

A. Wrapping Around Fingers

As shown in Fig. 5 (a), at time \( t_2 \) when pivoting ends, the knife’s handle has been lifted up and in contact with the palm. From \( t_2 \) to time \( t_3 > t_2 \), the joints of each of the finger 3, 4, 5 close in the proximal-to-distal order, in order to establish more contacts with the handle. A joint stops moving as soon as one of its descendent link collides with the handle, the joint reaches its limit, or it is constrained by other already fixed joints.

The knife is stabilized by the thumb and index finger at the antipodal positions on its handle. With the palm and the extra three fingers engaged in contact, there is little need for torisal torques at \( p_3 \) and \( p_2 \) to balance the knife’s rotation along the antipodal line \( \ell^* \). Any motion of the knife relative to the hand can now be prevented.

B. Caging

Next, the thumb and the index finger will be relocated. They are first detached from the handle. During the action, the palm and the rest of the fingers need to prevent the knife from sliding out of the hand. The knife does not need to be completely caged due to assistance from the gravity and friction.

Fig. 5: Finger gaits and hand caging leading to a power grasp. (a) End of pivoting; (b) wrapping around the knife’s handle; (c) reorienting the hand and knife; (d) removing the thumb and index finger; (e) moving the palm towards the knife plane of symmetry; (f) Closing in the thumb and index finger.

As shown in Fig. 6, the palm frame \( \{H\} \) is situated at the palm’s center of mass \( p_h \) with its \( y_{H} \)-axis aligned with the finger’s common joint axis direction when all the joint angles are zero, and \( z_{H} \)-axis perpendicular to the palm inner plane and pointing outward from back of the palm. Denote by \( \xi \) the angle between the vector \( \mathbf{p}_h - \mathbf{p}_0 \) and the table plane. For a general knife handle shape,

\[\text{Exertion of a large amount of such torque could also exceed the joint actuator limit.}\]
all the contact forces should stay inside their friction cone if \( \xi < \tan^{-1} \mu \). To achieve such configuration, from time \( t_3 \) to some time \( t_4 > t_3 \), the hand rotates about the \( y_W \)-axis of the world frame \( \{W\} \) with a certain angle such that the corresponding rotation matrix \( R_e \) satisfies the following constraint:

\[
\tan(\xi(t_4)) = \frac{\hat{z}^T R_e p_p(t_4)}{||p_p(t_3) - \hat{z} \hat{z}^T R_e p_p(t_3)||} < \mu.
\]

Since the motion of the knife in other directions are either constrained by the palm and fingers geometrically or prevented by gravity (conservation of the energy), we see that without changing the finger tip position, the knife cannot escape from the hand. For a thorough study of the caging, we refer to [36] and [37].

C. Power Grasp

As shown in Fig. 5 (d), the thumb and index finger are removed after the ‘caging’ configuration, during time period \( [t_5, t_6] \).

Let rotation matrix \( R_h \) represent the orientation of the palm frame \( \{H\} \) and \( p_5 \) be the center of mass of the little finger’s distal link. From time \( t_5 \) to \( t_6 > t_5 \), the palm is reoriented around \( p_5 \) to achieve a final orientation \( R_h(t_6) = R(t_2) \), which is aligned with the knife frame \( \{B\} \). The palm trajectory is then represented as \( R_h(t) \) and

\[
p_h(t) = R_h(t)^T p_5(t_4), \quad t \in (t_5, t_6),
\]

where

\[
R_h(t_4)^T (p_5(t_4) - p_h(t_4))
\]

is the position of little finger tip center relative to the palm frame \( \{H\} \) at time \( t_4 \). In the meantime, the finger 3, 4, 5 have their tip positions maintained while joint angles changing passively to accommodate the palm motion. It is depicted in Fig. 5 (d)–(e). Finally, the thumb and the index finger wrap around the knife’s handle along some predefined trajectory for the hand to achieve a power grasp configuration (Fig. 5 (f)).

V. SIMULATION

Simulation is conducted with the right-hand model of the Shadow Dexterous Hand E-Series and the UR10 arm manipulating a variety of knives in different scenarios. The two wrist joints of the Shadow hand are treated as part of the arm. A 6-axis force/torque sensor is mounted between the arm and hand. We assume that all the joints are torque driven with available joint angle and velocity readings.

The mass densities of the knife’s blade and handle are set to be 8000 kg/m³ and 600 kg/m³, respectively. The mass properties of the knife are obtained from its shape. Different scenarios are generated with the coefficients of friction ranging from 0.5 to 2 for the knife-finger contact and from 0 to 0.1 for the knife-board contact. The torsional torque coefficient ranges from 0.005 to 0.05.

All simulations are performed with the Mujoco physics engine [38]. Mujoco is a multibody dynamic simulator developed using a generalization of the Gauss least-square principle in generalized coordinates. The soft finger contact is modeled as a pyramidal friction cone coupled with a torsional torque opposing any rotation around the contact normal. Couplings among finger joints are described by equality constraints over their angles. The time step of simulation is set to be 2ms.

All the plots shown in the paper and the accompanying video were generated by controllers running at the frequency of 500Hz. The controller gain parameters used in the simulation were all diagonal matrices with equal entries on the main diagonal. The values of such entries are shown in the table below: The algorithm was always successful in lifting the knife up from the cutting board and achieving the power grasp configuration.

<table>
<thead>
<tr>
<th>Controller gain parameters</th>
<th>( K_{a,p} )</th>
<th>( K_{a,d} )</th>
<th>( K_{f,p} )</th>
<th>( K_{f,d} )</th>
<th>( K_{f,a} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_{a,p} )</td>
<td>1000</td>
<td>20</td>
<td>100</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>( K_{f,p} )</td>
<td>500</td>
<td>100</td>
<td>100</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

VI. DISCUSSION

The presented strategy for gripping a lying kitchen knife is inspired from the action of a human hand, which naturally minimizes the effort (by leveraging the cutting board’s support) and swiftly applies finger gaits to reach a power grasp on the knife’s handle. All the intricacies of the human hand movements that comprise this single skill are at present beyond full replication or automation. Nevertheless, close interleaving of continuous control policies and discrete topological transitions is a promising path to human-level dexterity.

Our strategy has overlooked several issues: slip at the knife’s tip, kinematics of finger contacts on the handle, knife stabilization against horizontal swaying, etc. While raising the knife’s handle, the hand often eases effort by allowing its tip to slide a little on the cutting board. Modeling of such slip can still treat the normal contact force as a variable and the frictional force as a dependent under Coulomb’s law, with necessary analysis of slip and stick modes. For finer finger gaits, we also need to consider kinematics of contact [39], [40] between the fingertips and the handle, and incorporate...
them into the system dynamics and controller design as in [12]. We can apply impedance control to stabilize the knife while it is being raised and grasped. A key issue is to curb the increasing complexity of the system dynamics.

To have its applicability the strategy needs to be validated through experiments, which is currently subject to the unavailability of an anthropomorphic hand and a 6-DOF Arm in our lab. Meanwhile, our goal will remain as to progressively understand and map more human hand skills to algorithms that the robotic hand can execute.

REFERENCES


